

I- Calculate the following integrals: (**35pts-5pts each**)

$$1) \qquad \int \frac{3^x + 4^x}{5^x} dx$$

$$2) \qquad \int \frac{dx}{\left(4-x^2\right)^{\frac{3}{2}}}$$

$$3) \qquad \int \frac{\ln x}{x^5} \, dx$$

4)
$$\int x \cos x \sin x dx$$

5)
$$\int (\tan x)^5 dx$$

$$6) \qquad \int \frac{x^5 + 1}{\left(x^3\right)\left(x + 3\right)} dx$$

$$7) \qquad \int \frac{3}{4+x^{\frac{1}{3}}} dx$$

II- Find the following derivatives: (10pts-5pts each)

1)
$$y = x \sec^{-1} x - \sqrt{1 - x^2} + 2\sqrt{x - 1} \sec^{-1} \sqrt{x}$$

2)
$$y = \left(\frac{2x4^x}{\sqrt{x^2+1}}\right)^3 (\tan x)^2 \frac{(5x+4)^3}{(2x-1)^2}$$

- III- Solve the following: (10pts-5pts each)
- 1) Using the shell method, find the volume of the solid generated by revolving about the y-axis, the region bounded by: $y = x^2 \& y = \sqrt{x}$.

2) Find the length of the curve
$$y = \ln(\sec x)$$
 from $x = 0$ to $x = \frac{\pi}{4}$

IV_ Solve for *x*: (10pts-5pts each)

1)
$$(\ln x)^3 - 5(\ln x)^2 + 6\ln x = 0$$



2)
$$\frac{e^{5x+4}}{e^{3x-2}} = e^{2x+4}$$

V- (**10pts-5pts each**)Test the following integrals for convergence. If they converge, find their limits:

a.
$$\int_{0}^{\infty} x(1+x)^{-5} dx$$

b. $\int_{1}^{\infty} \frac{1}{x(\sqrt{\ln x} + \ln^{2} x)} dx$

VI- (5pts) Solve for x when
$$\sin\left(\tan^{-1}\frac{x}{\sqrt{x^2+1}}\right) = \frac{2}{6}$$

VII- (10pts-5pts each) Evaluate the following integrals:

a)
$$\int_{0}^{e} \int_{0}^{e} \int_{0}^{e} (\ln x \ln y \ln z) dz dy dx$$

$$\int_{0}^{1} \int_{0}^{1} \int_{0}^{\frac{x}{y}} dy dx$$

$$\int_{0}^{1} \int_{0}^{1} \int_{0}^{\frac{x}{y}} dy dx$$

VIII- Solve the following problems: (10pts-5pts each)

a. Find the acute angle between the 2 vectors:

$$\vec{u} = 2\vec{i} - \vec{j}$$
 & $\vec{v} = 4\vec{i} + \frac{3}{2}\vec{j}$

b. Find the unit vector(s) that are parallel and normal to the vector $\vec{v} = \vec{i} - 4\vec{j}$